

# Fully stabilized 750-MHz Yb: fiber frequency comb

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**Abstract:** This study focuses on presenting a fully stabilized, self-referenced Yb: fiber frequency comb respectively phase locked to a microwave standard and an optical reference employing the highest, fundamental repetition rate of 750-MHz without additional external amplifiers and compressors. In addition, the challenge of phase locking the carrier envelope offset frequency for this high-repetition-rate fiber frequency comb is separately investigated in two schemes, namely,  $f$ - $2f$  self-referencing and an approach of phase locking a beat note between the Yb: fiber frequency comb and a continuous wave laser.

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**OCIS codes:** (320.7090) Ultrafast lasers; (140.3510) Lasers, fiber; (120.3930) Metrological instrumentation.

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## 1. Introduction

Over the last two decades, the emergence of optical frequency combs based on mode-locked femtosecond pulse lasers has prompted an era of tremendous scientific and technological breakthroughs in optical science and metrology [1,2]. Increasing the comb tooth spacing (equal to the laser's repetition rate) may permit the manipulation of each individual comb line and the utilization of a high power per mode to substantially improve the signal-to-noise ratio (SNR). High-repetition-rate ( $\geq 1$  GHz) frequency combs are rapidly gaining ground for applications in fields as diverse as optical metrology and high-resolution spectroscopy [3].

Previous literatures have reported the demonstrations of high-repetition-rate femtosecond mode-locked lasers with a repetition rate of  $> 1$  GHz. Classically, Ti: sapphire lasers were able to reach up to 10 GHz repetition rate and a high peak power, however they have disadvantages including the use of expensive and cumbersome green-pump lasers [4]. Recently, various improvements in laser technologies have led to remarkable improvements in high-repetition-rate mode-locked lasers. For example, Endo *et al.* demonstrated a 15-GHz spacing optical frequency comb based on a Kerr-lens mode-locked Yb:Y<sub>2</sub>O<sub>3</sub> ceramic laser with a quantum dot based semiconductor saturable absorber mirror as the mode-locker [5]. Additionally, another 10-GHz Er:Yb:glass mode-locked laser was also demonstrated [6].

The development of the stabilized frequency combs involves the detection and stabilization of two degrees of freedom of the comb, the repetition rate  $f_{\text{rep}}$ , and carrier envelope offset (CEO) frequency  $f_{\text{ceo}}$ . Thus, it is possible to precisely determine and express any of the comb teeth as the  $f_N = f_{\text{ceo}} + N \times f_{\text{rep}}$  (where  $N$  denotes the integer of mode number). Typically, it is more challenging to detect and phase lock the CEO frequency, particularly for the frequency comb with the high repetition rate. The  $f$ - $2f$  self-referencing approach is based on a coherent octave-spanning supercontinuum (SC), in which high pulse energy and short pulses are the key factors. It is feasible to use additional amplifiers and compressors; however, this is accompanied by the presence of new issues such as sensitivity to the environmental instability and system complexity. Therefore, researchers are more

interested in developing femtosecond lasers operating at a fundamental repetition rate of approximately 1 GHz by considering the tradeoff between the high repetition rate and the available power which is required for directly self-referenced  $f_{\text{ceo}}$  detection [7,8]. Previous studies demonstrated a fully phase-stabilized frequency comb in a diode-pumped Yb:CALGO solid-state laser and a monolithic solid-state laser [9,10]. On the other hand, fiber lasers are now capable of delivering pulse energies as high as those of solid-state lasers in addition to their key advantages such as compactness, low cost, passive cooling, and turn-key operability. Historically, ultrafast fiber lasers have been extensively involved in the areas of optical frequency metrology applications since 2003 [11–15]. Erbium (Er) fiber lasers with the GHz or multi-GHz repetition rate were reported in literatures [16,17]. Specifically, Ytterbium (Yb) fiber frequency comb is prominent due to its exceptional performances including superior power scalability and attainable Si-photo detector measurement. The possibility of highly-doped Yb: fibers fuels the miniaturization of cavity dimensions, and this is crucial for a high-repetition-rate frequency comb. To date, several demonstrations of Yb: fiber oscillators were reported in multiple configurations [18–23].

To the best of the authors' knowledge, the present study is the first demonstration of a self-referenced fiber frequency comb which utilizes the direct output from an Yb: fiber oscillator with the highest, 750-MHz fundamental repetition rate. Two different  $f_{\text{ceo}}$  stabilization schemes, namely, the self-referencing method that uses the higher oscillator's output power without extra amplifiers and an approach that phase locks a beat note between Yb: fiber comb and a continuous wave (CW) laser for a modest output power, were experimentally investigated. In addition, phase locking to the optical reference was performed. Compared with the scheme of phase locking to the radio frequency (RF) standard, the relative stability for every comb tooth of the 750-MHz Yb: fiber frequency comb improved by three to four orders of magnitude due to the stabilization of the optical reference. It is expected that this developed fiber frequency comb will be vital for different types of promising applications such as metrology and astronomical optical combs.

## 2. Experiment and results

The proposed 750-MHz frequency comb was based on a ring-cavity Yb: fiber oscillator with a similar configuration described in a previous study [24]. Nonlinear polarization evolution was employed to initiate mode-locking at a fundamental repetition rate of 750-MHz. The dispersion was compensated by a transmission type grating pair, and the laser operated at the stretched pulse mode locking regime with the center wavelength of 1045 nm. The maximal spectral bandwidth at the full-width-half-maximum (FWHM) was close to 27 nm, which represents the Fourier transform limited pulse duration of 70 fs given the assumption of a Gaussian profile. Highly stable mode-locking could be kept even at a maximum average output power of 900 mW under bi-directional pumping with 976 nm laser diodes operating at total power of 2.6 W. An intra-cavity piezoelectric transducer (PZT) and an electro-optic modulator (EOM) were incorporated as actuators for further phase locking systems. In the following parts, demonstrations of fully stabilized 750-MHz Yb: fiber frequency comb will be investigated separately from two aspects, phase locking to the RF standard and phase locking to the optical reference. Moreover, two different implements of stabilizing CEO frequency will be discussed in terms of phase locking to the RF standard.

### 2.1 Phase locking to the RF standard

#### 2.1.1 $f$ - $2f$ self-referencing scheme

Benefited from the peak power available from the developed oscillator, the generated pulse with a coupled pulse energy of  $\sim 1$  nJ was directly launched into a piece of tapered-photon crystal fiber (PCF) with a carefully designed dispersion profile [22] for the  $f_{\text{ceo}}$  detecting. The launched pulse duration was slightly negative chirped to  $\sim 120$  fs for compensating the normal

dispersion provided by a small section of single mode fiber spliced with the tapered-PCF at the input. The experimental configuration is schematically shown in Fig. 1. An octave-spanning spectral broadened SC spectrum was observed and plotted as shown in Fig. 2(a). The tapered-PCF output was collimated and sent into a Michelson-type  $f$ - $2f$  interferometer with a dichroic mirror (DM) to separate short and long wavelengths. The spectral components around 1200 nm were frequency doubled in a fan out-type periodically-poled lithium niobate (PPLN) crystal and overlapped in space and time with the spectral components at approximately 600 nm by using an optical delay line in an interferometer arm. The generated  $f_{\text{ceo}}$  was recorded by a Si avalanche photodiode detector (APD). The coherence of the one-octave SC spectrum is experimentally certified by the straightforward  $f_{\text{ceo}}$  detection with SNR  $> 40$  dB (at a resolution bandwidth (RBW) of 100 kHz) and a narrow linewidth of  $\sim 150$  kHz with a Lorentz fitting (at a 50 kHz RBW) as shown in Fig. 2(b). The  $f_{\text{ceo}}$  performance is typically sufficient for phase locking, and the reliability and reproducibility of the above-mentioned high coherence were confirmed under laboratory conditions.

As shown in Fig. 1, the detected  $f_{\text{ceo}}$  signal passed through a bandpass filter (BPF) and an RF amplifier prior to entering a 1/100 frequency divider. The phases were then compared with those of the RF synthesizer (SYN1) in a double balanced mixer, and the error signal was fed back to modulate the pump diode current through the loop filter. The first step in stabilizing the  $f_{\text{ceo}}$ , phase noise of both the free-running as well as the phase-locked  $f_{\text{ceo}}$  were investigated with a simple digital phase-measuring algorithm proposed in the study [25] (as shown in Fig. 2(c)). This measurement indicated that phase-locking the  $f_{\text{ceo}}$  to an external reference was achieved when the  $f_{\text{ceo}}$  feedback loop filter was activated, and this was demonstrated by the reduction in phase noise below approximately 15 kHz after stabilizing  $f_{\text{ceo}}$ . The corresponding residual integrated phase noise is calculated as 13.7 rad [26]. Meanwhile,  $f_{\text{rep}}$  was phase locked by directly comparing the phases between the detected  $f_{\text{rep}}$  signal and the second RF synthesizer (SYN2) and then by feeding back to the intra-cavity PZT actuator. The phase-stabilized  $f_{\text{ceo}}$  and  $f_{\text{rep}}$  were counted with a frequency counter (Pendulum CNT-90,  $\Pi$  type) at a gate time of 0.1 s, and the absolute Allan deviation of  $f_{\text{ceo}}$  and the relative Allan deviation of  $f_{\text{rep}}$  were calculated for all averaging times, respectively. The expected  $\tau^{-1}$  dependence of the stabilized  $f_{\text{ceo}}$  is observed in Fig. 3(a), and it shows a frequency stability ( $\delta f_{\text{ceo}}$ ) of 3.39 Hz at 1 s. Because the frequency range of counter (CNT-90) used for Fig. 3(b) is limited to 400 MHz, the frequency beat-down scheme was employed to evaluate the relative Allan deviation of  $f_{\text{rep}}$  ( $\delta f_{\text{rep}}/f_{\text{rep}}$ ). The measured beat-down frequency (the orange curve named " $f_{\text{rep}}$ " in Fig. 3(b)) is the frequency difference between the self-beat of the oscillator of  $f_{\text{rep}}$  and the RF synthesizer's output of 500 MHz. The plot of measurement limit (the purple curve in Fig. 3(b)) was achieved when replacing the oscillator's self-beat signal with a 750 MHz reference signal from another RF synthesizer. As shown in Fig. 3(b), the relative Allan deviation of the repetition rate is  $2.76 \times 10^{-11}$  at 1 s, and the simultaneously phase-locked  $f_{\text{rep}}$  exhibited a  $\tau^{-1}$  performance in the short term. As for the long term, though there is a deviation from the  $\tau^{-1}$  performance, it is subjected to the measurement limitation not the lock performance of the comb.

### 2.1.2 Phase locking a beat note scheme with a CW laser for $f_{\text{ceo}}$ stabilization

Self-referenced  $f_{\text{ceo}}$  detection for 750-MHz Yb: fiber frequency comb can be achieved without any amplifiers, however, a certain degree of output power from the oscillator is an indispensable condition. The present study also involved developing an alternative solution in the case of a modest output power of oscillator (lower than 500 mW) for this type of a high-repetition-rate fiber frequency comb. The experimental setup is shown in Fig. 4. The output of the Yb: fiber oscillator was combined with a CW laser MISER  $f_{\text{cw-MISER}}$  (monolithic isolated single-mode end-pumped ring laser with a free-running linewidth of approximately 3 kHz) at the fiber coupler to generate a beat note  $f_{\text{beat}}$  ( $f_{\text{beat}} = N \times f_{\text{rep}} + f_{\text{ceo}} - f_{\text{cw-MISER}}$ , and  $N$  is the integer of mode number). Phase locking was achieved by two phase-locked loops, namely

a loop acting on the PZT to stabilize the  $f_{\text{rep}}$  and another loop that controlled the pump power to lock  $f_{\text{ceo}}$  through the  $f_{\text{beat}}$ . The beat note observed directly from “ $f_{\text{beat}}\text{-PD}$ ” had a SNR of 40 dB (at a 100 kHz RBW), shown in Fig. 5(a). For the Fig. 5(b), every point represents the averaged linewidth from 3 to 4 times measurement after Lorentz fitting. Since the linewidth at RBW 20 kHz is a little broader than the linewidth at RBW 30 kHz, we roughly estimated the FWHM linewidth as  $\sim 70$  kHz at a 30 kHz RBW. Additionally, the CW laser was simultaneously stabilized to another in-house fabricated, fully stabilized 50 MHz Er: fiber frequency comb [15], and thus it behaved as an absolute frequency determined CW laser.

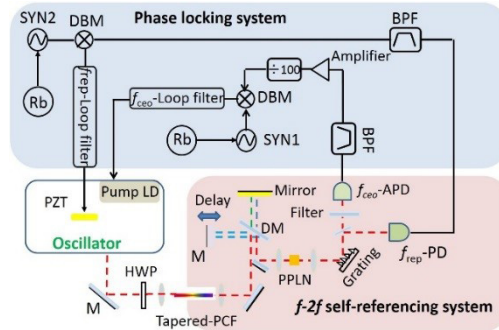


Fig. 1. A 750-MHz Yb: fiber frequency comb stabilized to an RF reference with the  $f\text{-}2f$  self-referencing scheme for  $f_{\text{ceo}}$  detection. SYN: synthesizer; DBM: double balanced mixer; BPF: band pass filter; APD: avalanche photodiode detector; DM: dichroic mirror; M: mirror; HWP: half wave-plate; PZT: piezoelectric transducer; LD: laser diode.

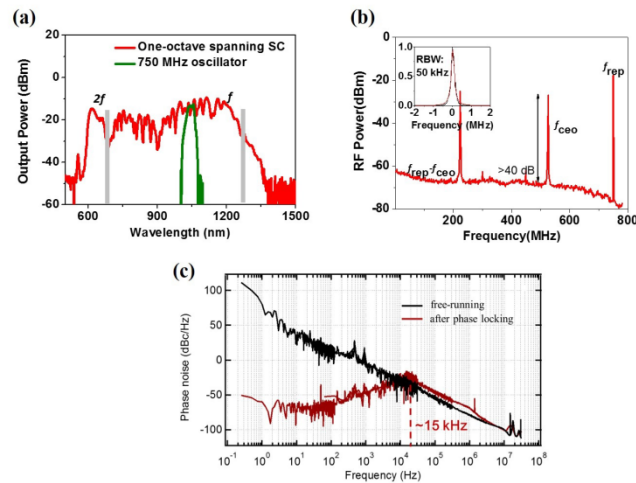


Fig. 2. (a) Octave-spanning SC spectrum generation by a tapered PCF. The indicated spectral portions at 600 nm and 1200 nm are used for the self-referencing approach (Green line: optical spectrum of 750-MHz Yb oscillator centered at 1045 nm) (b) Detection of free-running  $f_{\text{ceo}}$  signal. The inset denotes the linewidth  $f_{\text{ceo}}$  of 150 kHz with a Lorentz fitting at 50 kHz RBW. (c) Phase noise of the  $f_{\text{ceo}}$  (free-running: black; after phase locking: red).

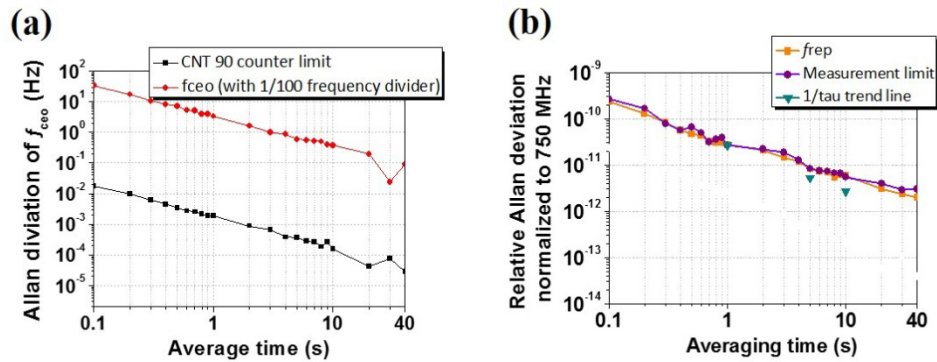


Fig. 3. (a) Absolute Allan deviation of  $f_{\text{ceo}}$  ( $\delta f_{\text{ceo}}$ ) (Red line: absolute Allan deviation with 1/100 frequency divider; Black line: CNT 90 counter limit). (b) Relative Allan deviation of  $f_{\text{rep}}$  ( $\delta f_{\text{rep}}/f_{\text{rep}}$ ) normalized to 750 MHz. (Orange line: relative Allan deviation of  $f_{\text{rep}}$ ; Purple line: Measurement limit; Green triangle plot: 1/tau trend points).

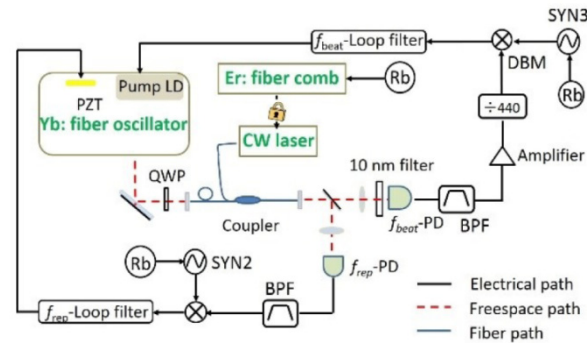


Fig. 4. A 750-MHz Yb: fiber frequency comb stabilized to an RF reference by phase locking the beat note  $f_{\text{beat}}$  scheme for  $f_{\text{ceo}}$  detection.

To investigate the stability characteristics of the frequency comb system proposed above that is liberated from generating an octave SC, a relative Allan deviation of  $f_{\text{rep}}$  and an absolute Allan deviation of  $f_{\text{beat}}$  were measured by a frequency counter (Agilent Model 53132A, measured with the internal trigger mode exhibiting  $\Lambda$  type), respectively. Both Allan deviation plots were calculated for all averaging times from the same data set taken with the gate time of 0.1 s, and it was evident both of them were subjected to the counter limit as shown in Figs. 5(c) and 5(d). The measured deviations exhibited the  $\tau^{-1/2}$  performance which one would expect for white frequency noise but not for phase noise. This is a result of employing the frequency counter ( $\Lambda$  type) which has dead times after each measurement [27]. Moreover, thanks to the high-repetition-rate comb, the absolute comb mode number  $N$  beating with MISER could be easily determined as  $N = 376168$  by using a standard low resolution wavemeter (ADVANTEST: Q8326; res: 100 MHz). Therefore, the absolute frequency of the comb modes was determined without ambiguity. It should be noted that the coherent link between two-color fiber combs (Yb: fiber comb and Er: fiber comb) with 15-times repetition rate separation ( $f_{\text{rep-Yb}} = 750\text{-MHz}$  and  $f_{\text{rep-Er}} = 50\text{-MHz}$ ) was demonstrated in this scheme.

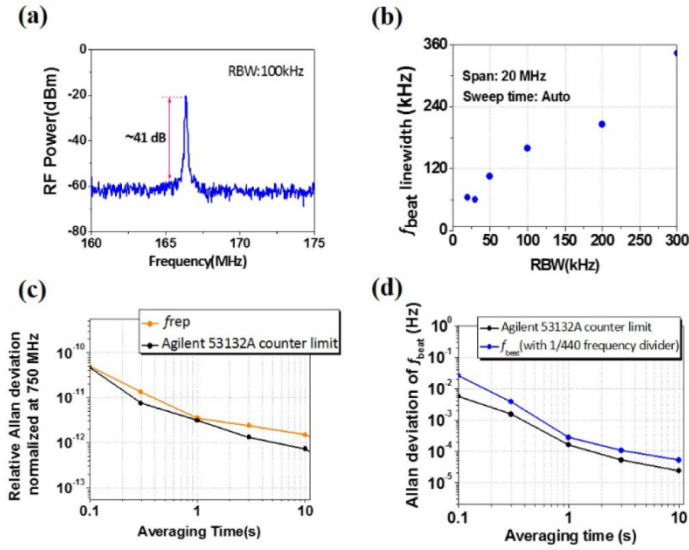


Fig. 5. (a) Free-running  $f_{\text{beat}}$  under the condition of phase-locked  $f_{\text{rep}}$ ; (b)  $f_{\text{beat}}$  linewidths against the different RBWs; (c) Relative Allan deviations of  $f_{\text{rep}}$  ( $\delta f_{\text{rep}}/f_{\text{rep}}$ ); (d) Absolute Allan deviation of  $f_{\text{beat}}$  ( $\delta f_{\text{beat}}$ ). All data were measured by a frequency counter (Agilent 53132A, internal trigger mode).

## 2.2 Phase locking to the optical reference

When the  $f_{\text{rep}}$  is stabilized to a microwave standard, the stability of every comb tooth is a multiplicative product of  $\delta f_{\text{rep}}$  with the vast integral mode number  $N$  ( $N \approx 10^5$ ). Therefore, as for the repetition rate, the calculated relative stability of  $2.76 \times 10^{-11}$  at 1 s in the RF domain ( $\delta f_{\text{rep}}$  normalized at  $f_{\text{rep}} = 750$ -MHz) corresponds to the relative stability of every comb tooth with  $2.76 \times 10^{-11}$  at 1 s in the optical domain ( $N \times \delta f_{\text{rep}}$  normalized at  $f_{\text{opt}} = 282$  THz in which  $f_{\text{opt}}$  denotes the optical frequency) as well. Alternatively,  $N \times f_{\text{rep}}$  could be effectively controlled via a heterodyne beat between a tooth of the optical comb and a stable CW laser in the optical domain. In this way, the  $N$  times effects could be readily eliminated since the relative stability of  $f_{\text{beat}}$  ( $\delta f_{\text{beat}}/f_{\text{opt}}$ ) actually represents the relative stability of  $N \times f_{\text{rep}}$  ( $N \times \delta f_{\text{rep}}/f_{\text{opt}}$ ) in considering of  $N \times f_{\text{rep}} \gg f_{\text{ceo}}$ .

Figure 6 shows a schematic explanation of the setup in which the 750-MHz Yb: fiber oscillator stabilized to an optical standards is employed. A small portion of the output of the 750-MHz Yb oscillator was combined with the CW laser (MISER) at the fiber coupler to generate  $f_{\text{beat}}$  in a manner similar to that employed in Fig. 4. The free-running  $f_{\text{beat}}$  has the same high quality of performance as that shown in Fig. 5(a). The phase locking system of  $f_{\text{beat}}$  includes a BPF (centering at 30 MHz), a 1/44 frequency divider, SYN3, and an in-house fabricated  $f_{\text{beat}}$  loop filter. As the fore-mentioned case is different from the case in which the 750-MHz Yb: fiber frequency comb is phase-locked to the RF reference (Fig. 4), an intra-cavity EOM assumed the role of the system actuator for  $f_{\text{beat}}$  due to its capability of high speed control. With respect to the  $f_{\text{ceo}}$ , the previously described self-referencing approach was implemented.

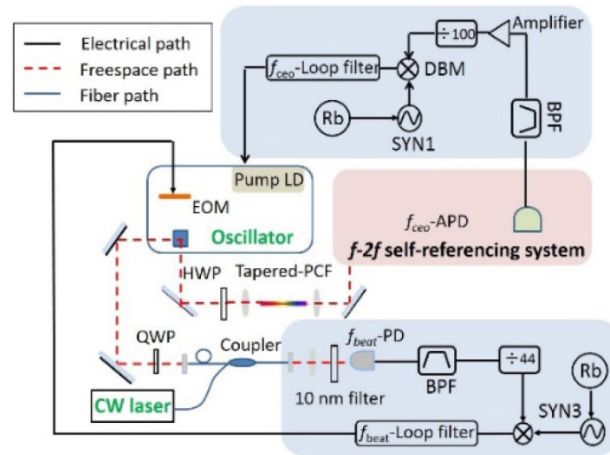


Fig. 6. A 750-MHz Yb: fiber frequency comb stabilized to optical reference.

As shown in Fig. 7(a), the noise analysis of the phase-locked  $f_{\text{beat}}$  was first conducted. It revealed a significant reduction in the stabilized  $f_{\text{beat}}$  phase noise by the feedback to the intra-cavity EOM, and this indicated the servo bandwidth of approximately 45 kHz, and the residual integrated phase noise of 4.6 rad. The limited servo bandwidth and the rather large residual phase noise are due to the severe resonating peaks around 800 kHz of the intra-cavity EOM used in present setup. In addition, as shown in Fig. 7(b), relative Allan deviations of the fully stabilized 750-MHz Yb: fiber comb to the optical reference were calculated from the acquired data by using the frequency counter (Pendulum, CNT-90, II type). The relative stability of the  $f_{\text{beat}}$  ( $\delta f_{\text{beat}}/f_{\text{opt}}$ ) is  $\sim 4.04 \times 10^{-15}$  at 1s (normalized to 282 THz and plotted by the blue curve). Finally, the comparison about the relative stability to the reference which contributes to the every comb tooth " $\delta f_N/f_{\text{opt}}$ " between two phase locking cases (phase locking to the RF standard and to the optical reference) was shown in Table 1. In terms of phase locking to the RF standard, the relative stability could be represented as " $(\delta f_N/f_{\text{opt}})^2 \sim (\delta f_{\text{rep}}/f_{\text{rep}})^2 + (\delta f_{\text{ceo}}/f_{\text{opt}})^2 (N \approx 10^5)$ "; For the case of phase locking to the optical reference, the relative stability could be expressed as " $(\delta f_N/f_{\text{opt}})^2 \sim (\delta f_{\text{beat}}/f_{\text{opt}})^2 + (\delta f_{\text{ceo}}/f_{\text{opt}})^2 (N \approx 10^5)$ ". The same self-referencing scheme is responsible for stabilizing the offset frequency, and thus the phase locked  $f_{\text{ceo}}$  (plotted by the red curve of Fig. 7(b)) exhibits almost the same stability as that shown in the Fig. 3(a). We should note that the relative stability " $\delta f_N/f_{\text{opt}}$ " mentioned here does not represent the absolute frequency stability of the comb modes, but indicates the relative (in-loop) stability to the reference whose absolute stability was determined by the reference. Here we could show that the relative stability of comb tooth improved by approximately three to four orders of magnitude when compared to that of the case stabilized to the RF standard ( $\sim 2.76 \times 10^{-11} \tau^{-1}$ ). In this study, the stabilization of the  $f_{\text{beat}}$  through the feedback on an EOM was realized with the help of a frequency divider due to the extremely severe resonating peaks of the EOM used. For further improvement in performances, the tight phase-locking and broader servo bandwidth are highly desirable and could be potentially achieved by optimizing mechanical design of the holder of intra-cavity EOM to suppressing the resonating peaks and in-house fabricated  $f_{\text{beat}}$  loop filter.



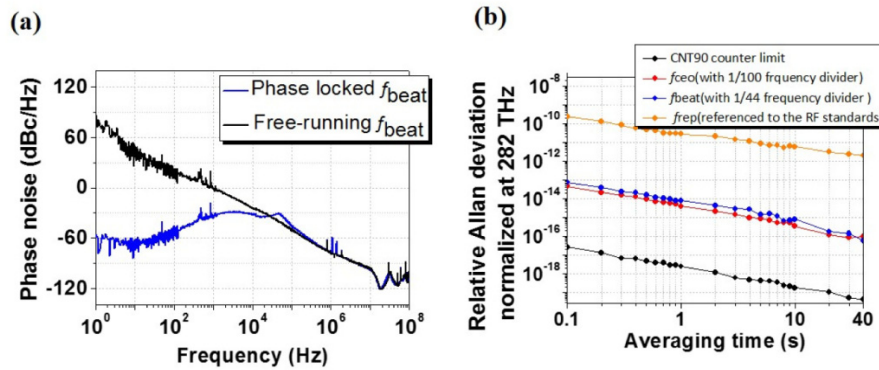


Fig. 7. (a) Phase noise of the  $f_{\text{beat}}$  (free-running: black; after phase locking: blue); (b) Relative Allan deviations of  $f_{\text{beat}}$  (the blue curve),  $f_{\text{cco}}$  (the red curve) normalized to the optical standard of 282 THz; Black curve denotes the counter limit. Relative stability of the  $f_{\text{rep}}$  ( $\delta f_{\text{rep}}/f_{\text{rep}}$ , the orange curve) referenced to the RF standards (the plot is the same as that shown in Fig. 3(b)) is also shown for comparison.

Table 1. Comparison for the relative stability of every comb tooth  $\delta f_n/f_{\text{opt}}$

Parameter	Relative stability of $f_{\text{opt}}$ @ 1 s
Method	
Phase locking to RF standard	$2.71 \times 10^{-11}$
Phase locking to optical reference	$9.05 \times 10^{-15}$

### 3. Conclusions

We have demonstrated an Yb: fiber frequency comb that is fully stabilized to the RF standard and an optical reference with the fundamental repetition rate of 750-MHz without an extra amplifier and a compressor. To the best of the authors' knowledge, this is the highest repetition rate of a self-referenced absolute frequency determined frequency ruler by using a direct oscillator output of a fiber comb. In addition, an alternative solution of  $f_{\text{cco}}$  detection was investigated under a modest average power, and the coherent link between two-color combs (Yb: fiber comb and Er: fiber comb) was realized in a 15-times repetition rate separation ( $f_{\text{rep-Yb}} = 750\text{-MHz}$  and  $f_{\text{rep-Er}} = 50\text{-MHz}$ ) with a narrow mode linewidth of 70 kHz (at a 30 kHz RBW). Of importance, by employing the EOM as an actuator, phase locking to an optical reference was achieved and showed that, the relative stability of every comb tooth ( $\sim 9.05 \times 10^{-15} \tau^{-1}$ ) improved by three to four orders of magnitude when compared to that in the case of the RF standard ( $\sim 2.71 \times 10^{-11} \tau^{-1}$ ). Future work will include the optimization of the EOM mechanical design and further enhancing the environmental stability of the system. It is expected that this compact, fully stabilized Yb: fiber frequency comb developed in this study will play a significant role in diverse applications.

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