

TECHNICAL REPORT

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Two Algorithms for Finding a Near-Maximum Clique*

(Preliminary version)

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We present two algorithms for finding a near-maximum clique Q_{\max} in a given graph $G=(V,E)$ that are based upon a simple algorithm for finding an exactly maximum clique Q_{\max} in [TKT]. The first algorithm NMCLIQ works in $O(n^3)$ time where n is the number of vertices of G , while the second is not so. These were evaluated experimentally for several graphs on SONY NEWS-830(68020,16MHz) in C. This report is described throughly based upon [TKT].

The first algorithm is as follows.

```

procedure NMCLIQ(G)
    {Graph G=(V,E)}
begin
    Q:= $\phi$  {global variable Q is to constitute a complete subgraph}
    arrange the vertices of V in nondecreasing order
        with respect to their degrees
    NUMBERING(V,No)
    NEXTEND(V, No)
    procedure NEXTEND(SUBG, No)
    begin
        q:=vertex in SUBG such that No(q) is the greatest
        Q:=Q $\cup$ {q}
        SUBGq:=SUBG $\cap$  $\Gamma$ (q)
        if SUBGq:= $\phi$ 
            then {Q is a clique}
                Qmax:=Q fi
            else
                arrange the vertices of SUBGq in nondecreasing order
                    with respect to their degrees
                NUMBERING(SUBGq, NewNo)
                NEXTEND(SUBGq, NewNo)
            fi
        end {of NEXTEND}
    end {of NMCLIQ}

```

Note here that one call of NEXTEND(SUBG,No) without its recursive call takes $O(|SUBG|^2)$ ($|SUBG| \leq n$) and that the maximum number of calls of NEXTEND is at most n . Then the algorithm NMCLIQ(G) works in $O(n^3)$ time.

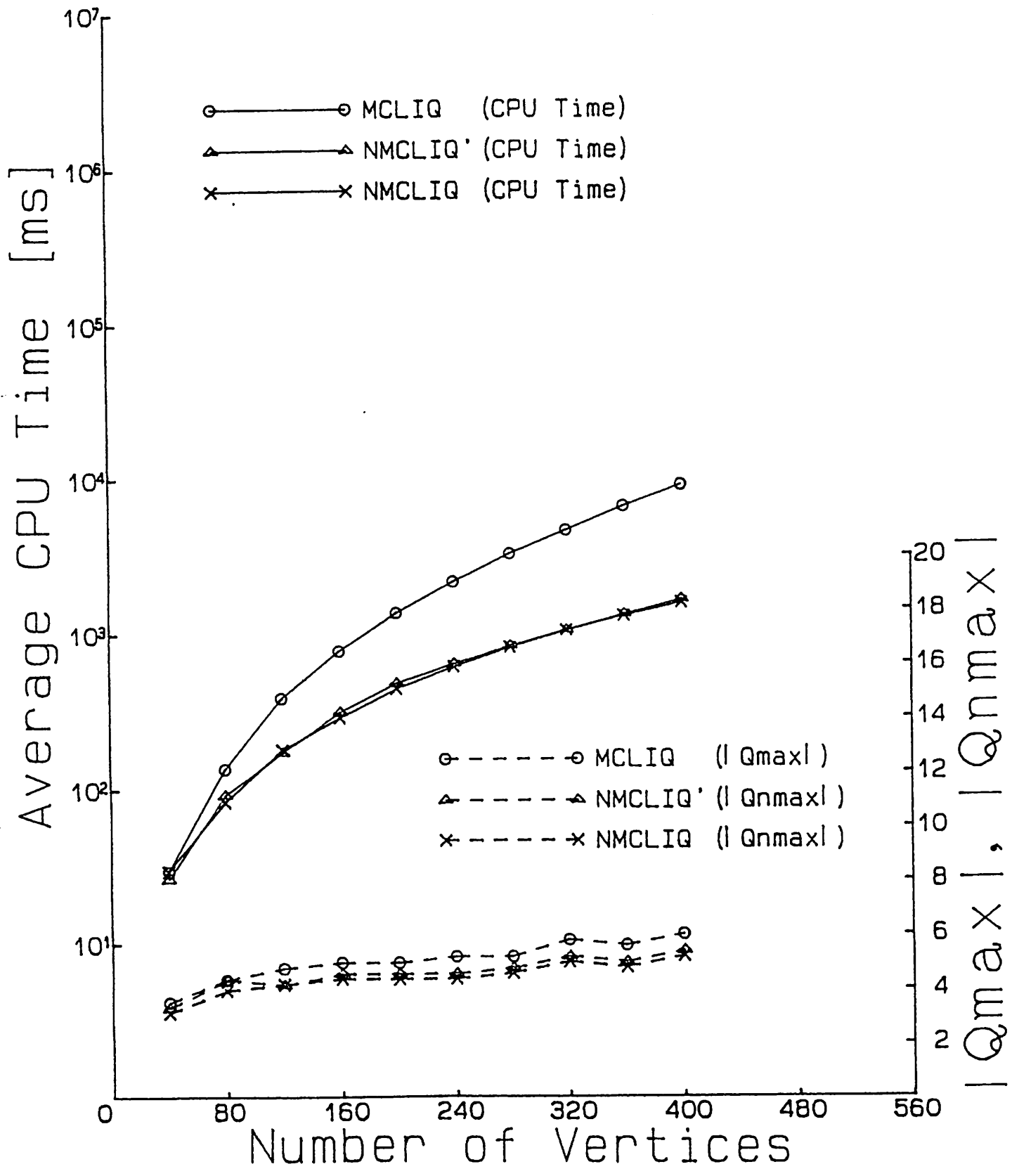
The second algorithm is as follows.

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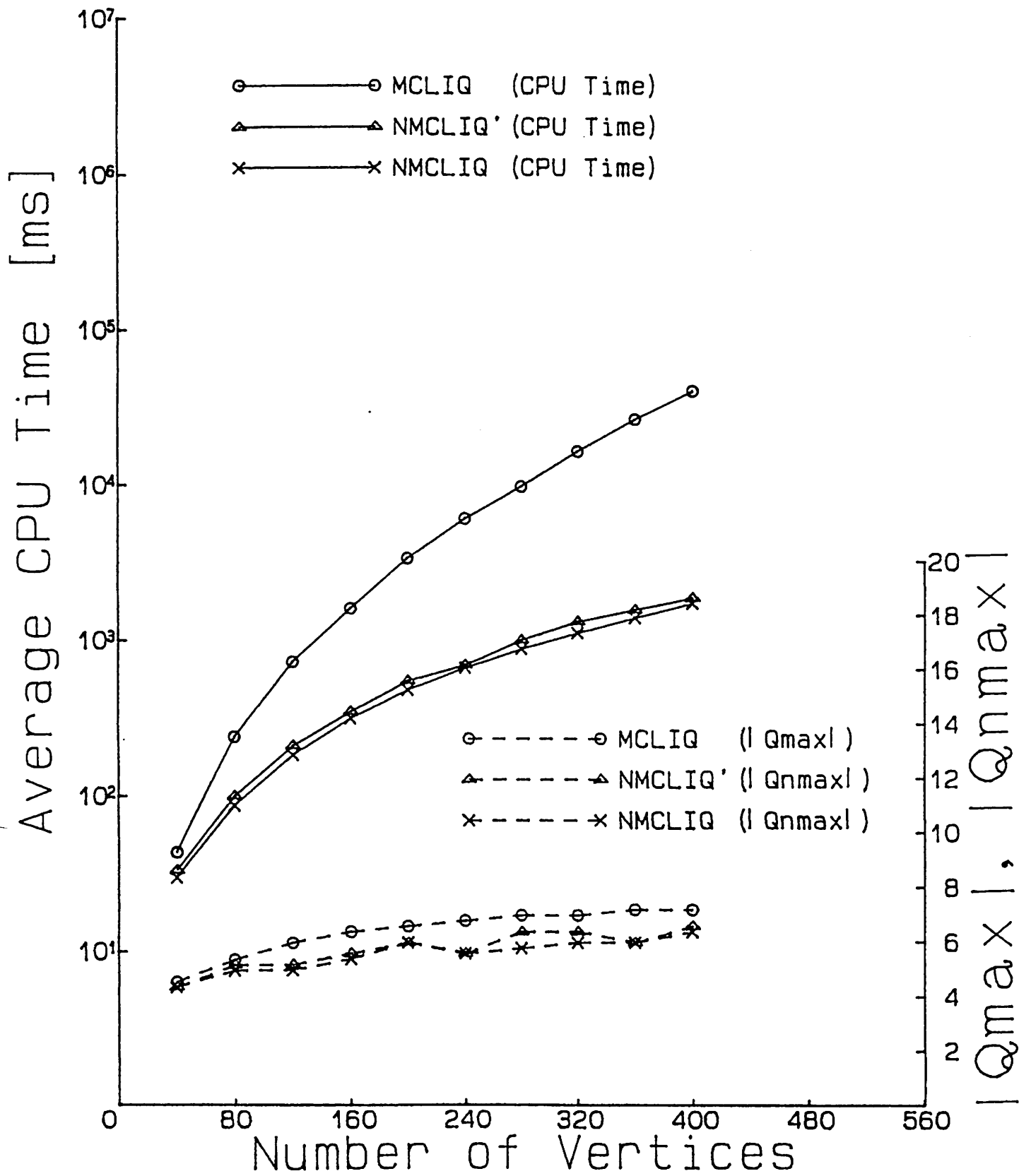
procedure NMCLIQ' (G)
    {Graph G=(V,E)}
begin
    Q:= $\phi$  {global variable Q is to constitute a complete subgraph}
    Qnmax:= $\phi$  {global variable Qnmax is a near-maximum clique having been
        found}
    arrange the vertices of V in nondecreasing order
        with respect to their degrees
    NUMBERING(V, No)
    NEXTEND(V, No)
    procedure NEXTEND(SUBG, No)
    begin
        Nmax:=Max{No(p) | p $\in$  SUBG}
        while SUBG $\neq \phi$ 
            do q:=vertex in SUBG such that No(q) is the greatest
                if No(q)< Nmax
                    then exit fi
                if |Q| +No(q) $\leq$  |Qnmax|
                    then { $\omega(Q \cup SUBG) \leq |Q_{nmax}|$ } exit fi
                Q:=Q $\cup$  {q}
                SUBGq:=SUBG $\cap \Gamma$  (q)
                if SUBGq:= $\phi$ 
                    then {Q is a clique}
                        if |Qnmax| < |Q| then Qnmax:=Q fi
                    else
                        arrange the vertices of SUBGq in nondecreasing order
                            with respect to their degrees
                        NUMBERING(SUBGq, NewNo)
                        NEXTEND(SUBGq, NewNo)
                    fi
                SUBG:=SUBG-{q}
                Q:=Q-{q}
            od
        end {of NEXTEND}
    end {of MCLIQ'}

```

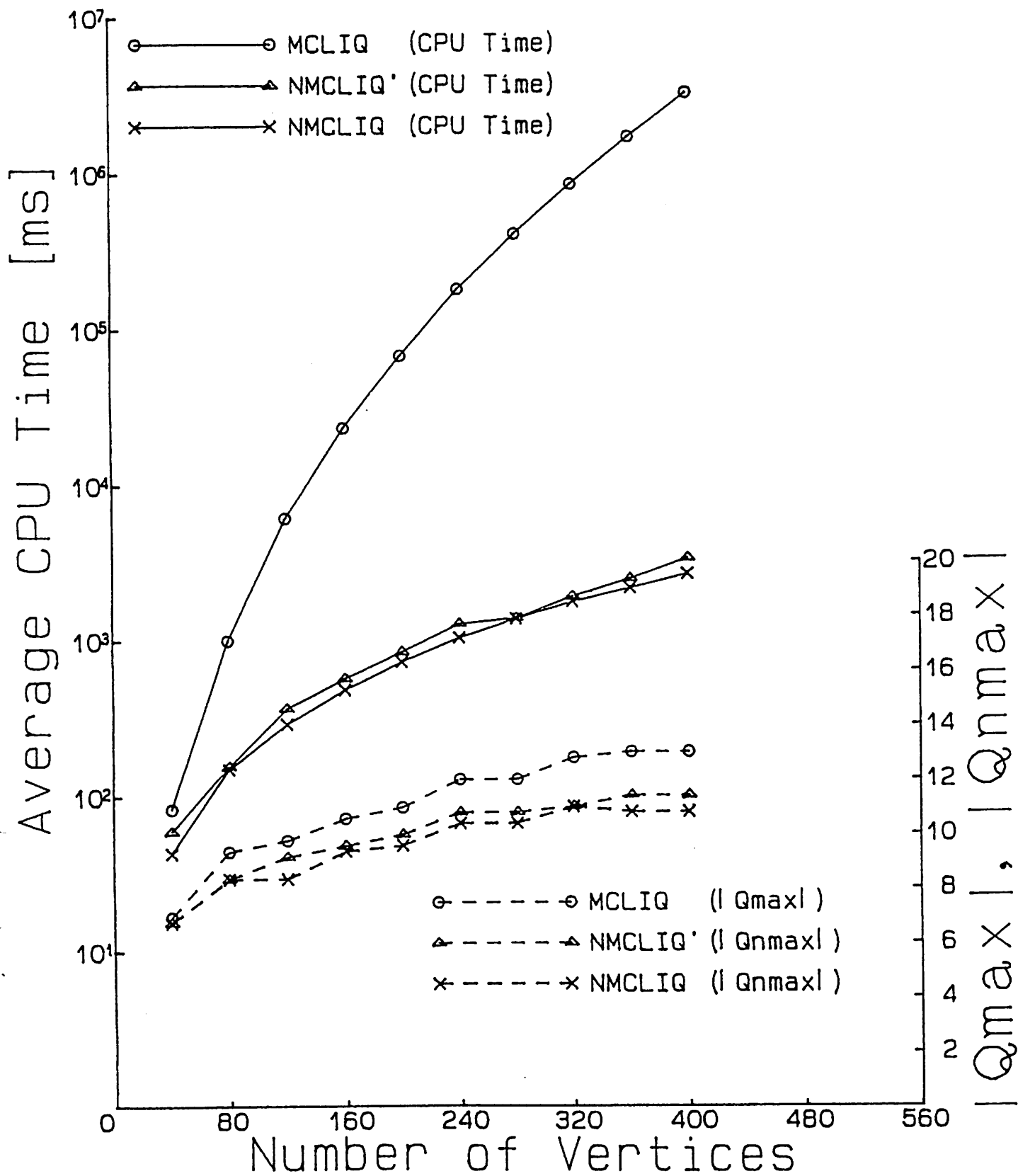
Now the experimental results follow.



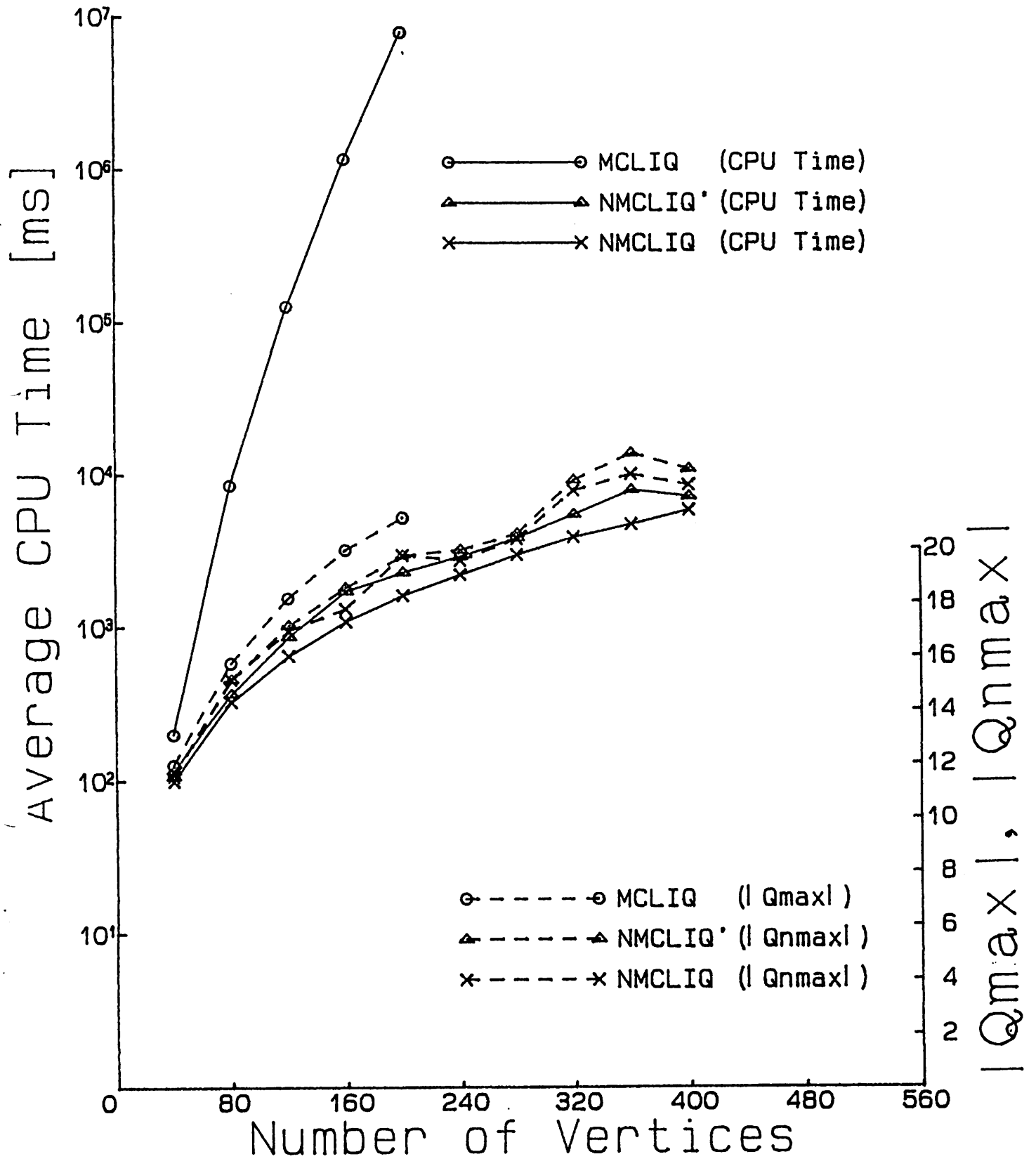
Graphs of Density = 0.15



Graphs of Density = 0.25



Graphs of Density = 0.50



Graphs of Density = 0.75

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REFERENCE

[TKT] E.TOMITA, Y.KOHATA AND H.TAKAHASHI, "A Simple Algorithm for Finding a Maximum Clique", Technical Report, UEC-TR-C5, 1988.