Two Algorithms for Finding a Near-Maximum Clique

著者（英）	Etsuji Tomita, Shinsuke Mitsuma, Haruhisa Takahashi
journal or publication title	Technical Report of the University of Electro-Communications
volume	UEC-TR-C1
page range	1-9
year	1988
URL	http://id.nii.ac.jp/1438/00001900/

TECHNICAL REPORT
of UEC, UEC-TR-C1, 1988

Two Algorithms for Finding a Near-Maximum Clique*

(Preliminary version)

Etsuji Tomita Shinsuke Mitsuma Haruhisa Takahashi

Department of Communications and Systems

THE UNIVERSITY OF ELECTRO-COMMUNICATIONS

Chofugaoka, Chofu, TOKYO 182 JAPAN

We present two algorithms for finding a near-maximum clique Qnmax in a given graph G=(V,E) that are based upon a simple algorithm for finding an exactly maximum clique Qmax in [TKT]. The first algorithm NMCLIQ works in $O(n^3)$ time where n is the number of vertices of G, while the second is not so. These were evaluated experimentally for several graphs on SONY NEWS-830(68020,16MHz) in C. This report is described throughly based upon [TKT].

The first algorithm is as follows.

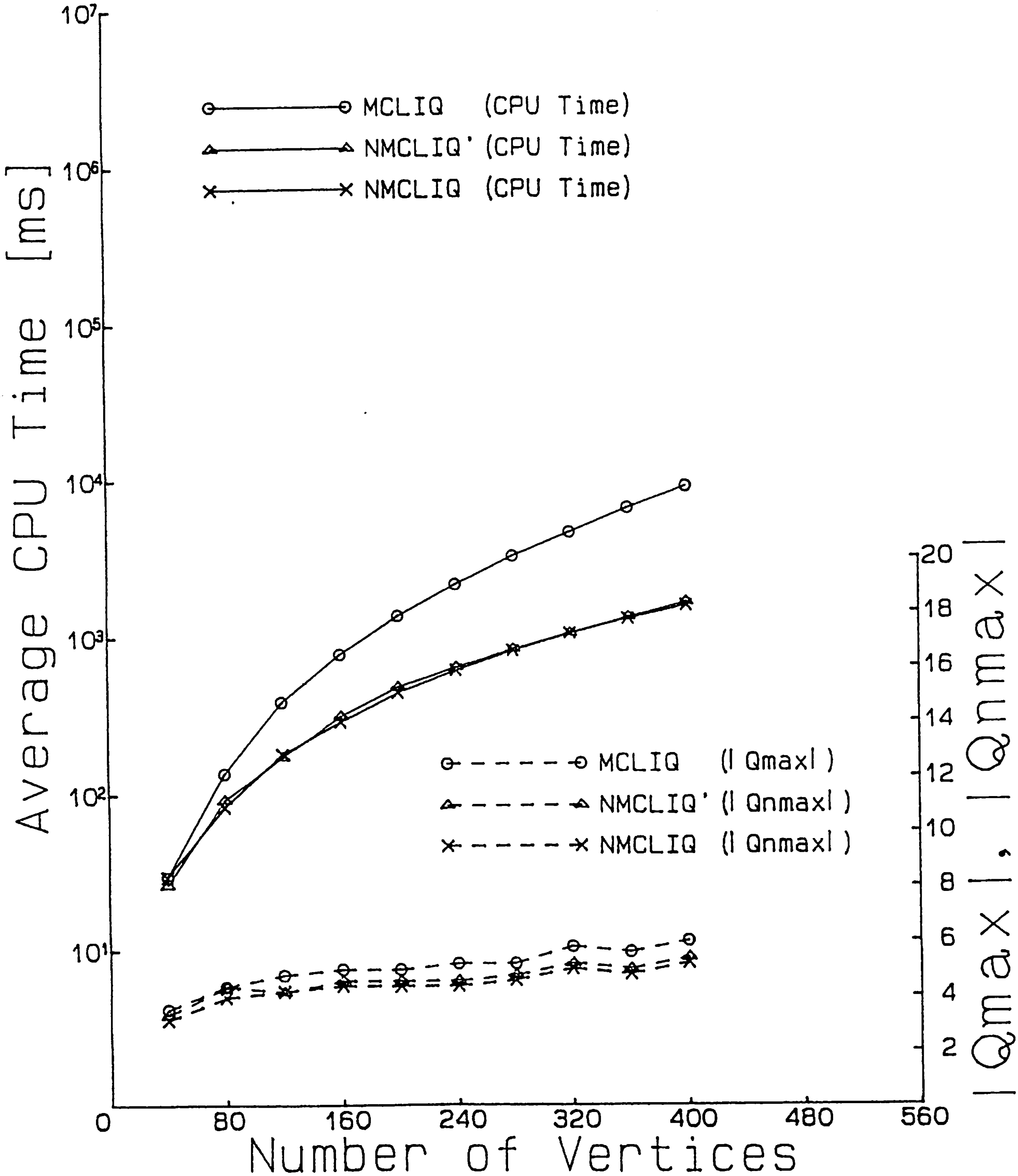
```
procedure NMCLIQ(G)
          {Graph G=(V,E)}
begin
  Q:=φ   {global variable Q is to constitute a complete subgraph}
  arrange the vertices of V in nondecreasing order
          with respect to their degrees
  NUMBERING(V,No)
  NEXTEND(V, No)
  procedure NEXTEND(SUBG, No)
  begin
      q:=vertex in SUBG such that No(q) is the greatest
      Q:=Q∪{q}
      SUBG_q:=SUBG∩Γ(q)
      if SUBG_q:=φ
         then {Q is a clique}
            Qnmax:=Q fi
         else
              arrange the vertices of SUBG_q in nondecreasing order
                     with respect to their degrees
              NUMBERING(SUBG_q, NewNo)
              NEXTEND(SUBG_q, NewNo)
      fi
  end {of NEXTEND}
end {of NMCLIQ}
```

Note here that one call of NEXTEND(SUBG,No) without its recursive call takes $O(|SUBG|^2)$ ($|SUBG \leq n|$) and that the maximum number of calls of NEXTEND is at most n. Then the algorithm NMCLIQ(G) works in $O(n^3)$ time.

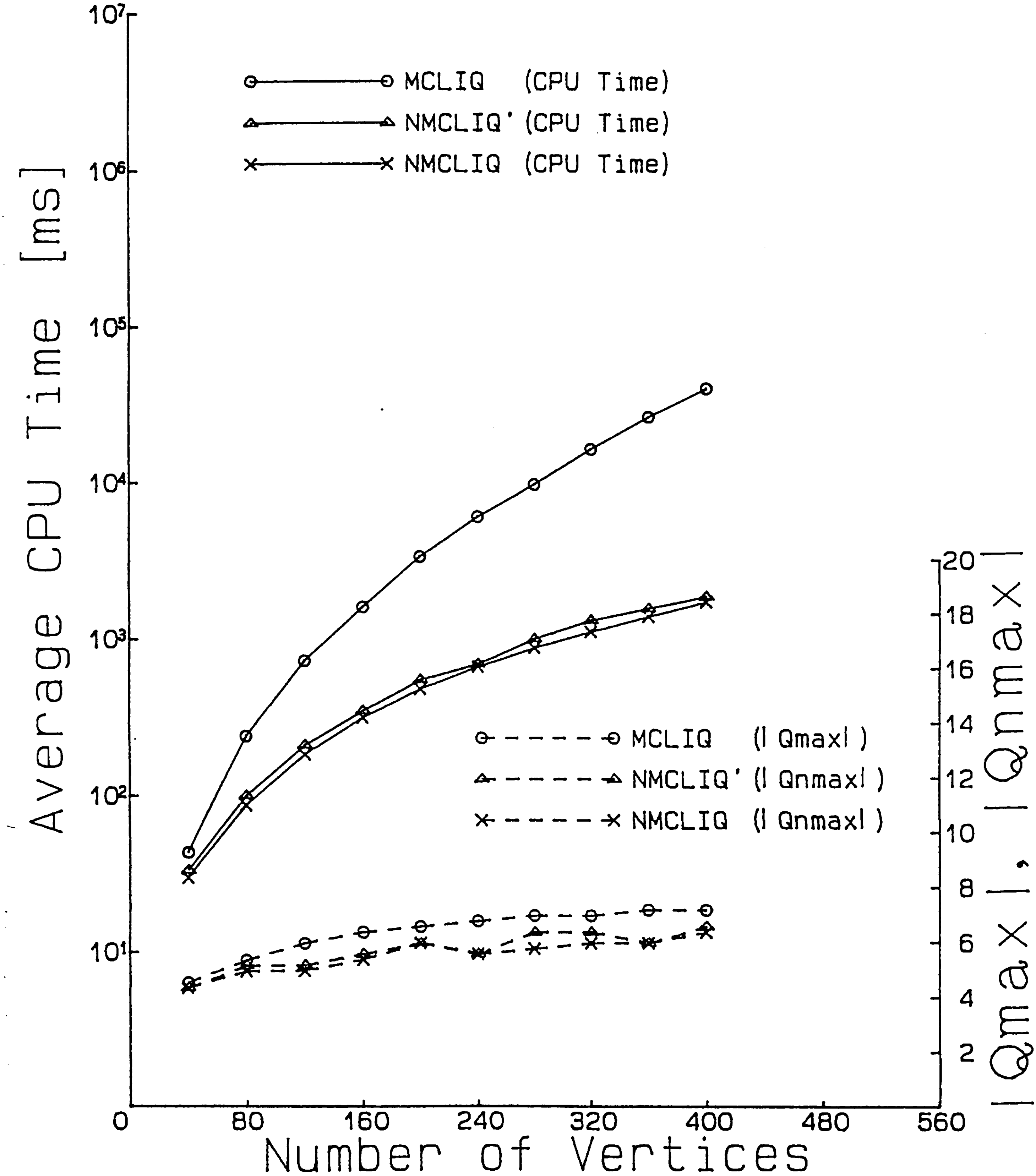
The second algorithm is as follws.

```
procedure NMCLIQ'(G)
          {Graph G=(V,E)}
begin
  Q:=φ   {global variable Q is to constitute a complete subgraph}
  Qnmax:=φ   {global variable Qnmax is a near-maximum clique having been
              found}
  arrange the vertices of V in nondecreasing order
          with respect to their degrees
  NUMBERING(V,No)
  NEXTEND(V, No)
  procedure NEXTEND(SUBG, No)
  begin
    Nmax:=Max{No(p) | p∈ SUBG}
    while SUBG≠ φ
       do q:=vertex in SUBG such that No(q) is the greatest
          if No(q)< Nmax
             then  exit fi
          if  |Q| +No(q)≦ |Qnmax|
             then {ω (Q∪SUBG)≦ |Qnmax| } exit fi
          Q:=Q∪{q}
          SUBGq:=SUBG∩Γ(q)
          if SUBGq:=φ
             then {Q is a clique}
                  if  |Qnmax| < |Q|  then Qnmax:=Q fi
             else
                  arrange the vertices of SUBGq in nondecreasing order
                          with respect to their degrees
                  NUMBERING(SUBGq, NewNo)
                  NEXTEND(SUBGq, NewNo)
          fi
          SUBG:=SUBG-{q}
          Q:=Q-{q}
       od
  end {of NEXTEND}
end {of MCLIQ'}
```

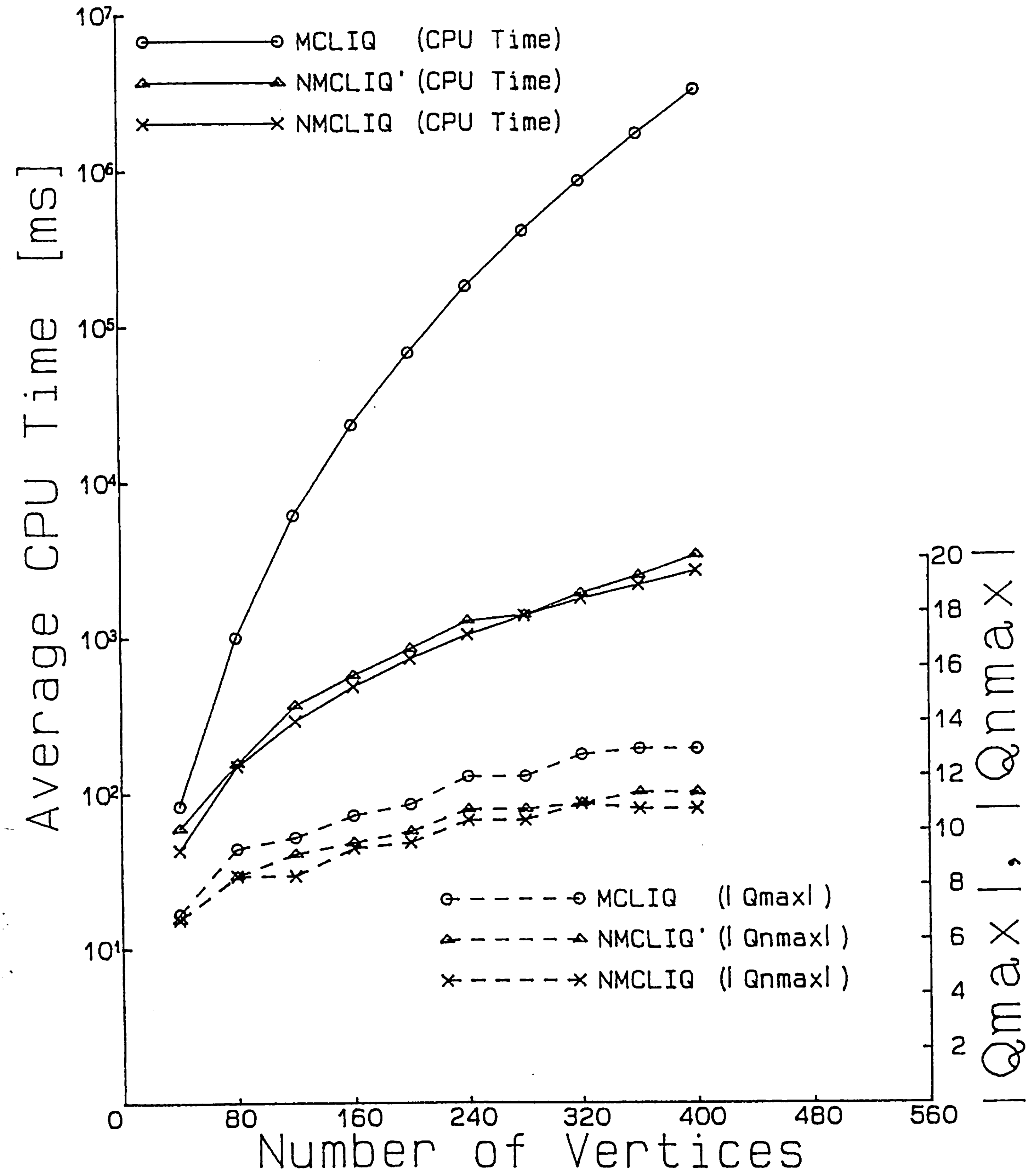
Now the experimental results follow.



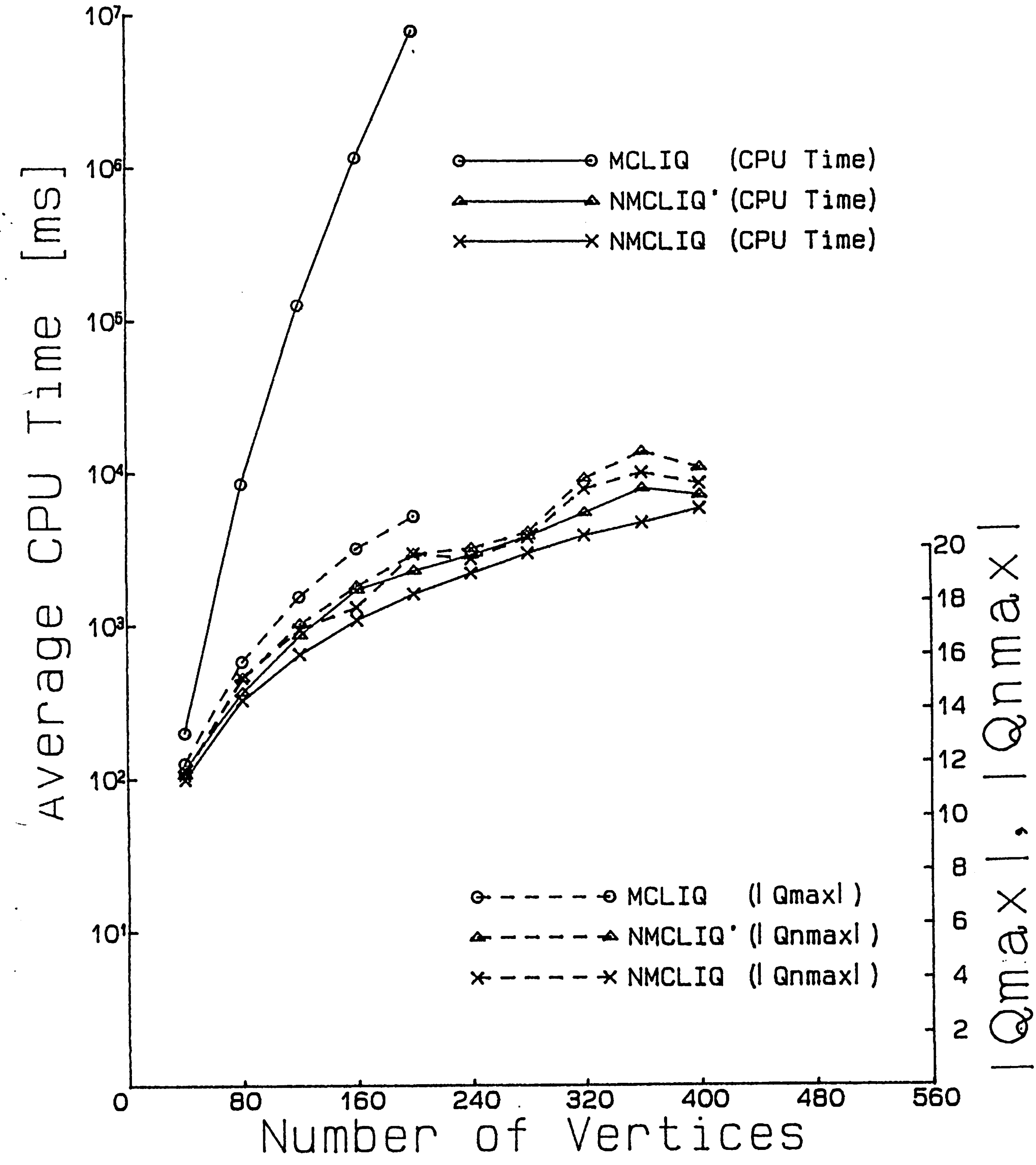
Graphs of Density = 0.15



Graphs of Density = 0.25



Graphs of Density = 0.50



Graphs of Density = 0.75

Acknowledgement The authors wish to express their gratitude to Yasuhiro Kohata for his useful comment.

REFERENCE

[TKT] E.TOMITA, Y.KOHATA AND H.TAKAHASHI, "A Simple Algorithm for Finding a Maximum Clique",Technical Report,UEC-TR-C5,1988.